The Application of Mathematical Modeling in the Teaching of Advanced Mathematics

Yi Liu[1], Xiaobo Liu[2,*]

[1] Ideological and political theory teaching and research Department, Jiangsu Police Institute, Nanjing, China

[2] School of information engineering, Nanjing Xiaozhuang University, Nanjing, China

*Corresponding author: Xiaobo Liu

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Abstract: Combining the practice of higher mathematics classroom teaching, and the problem-oriented teaching, through the actual case analysis, how to integrate the ideas of mathematical modeling into the teaching of higher mathematics, and improve students' interest and quality of learning are analyzed in this paper.

1. Introduction

In the teaching of higher mathematics, there is a phenomenon of excessive pursuit of rigor, which leads students to see Abstract concepts and theorems and affect the initiative and enthusiasm of students. In order to change this phenomenon, the idea of mathematical modeling should be incorporated into the teaching and be closed to the actual life to improve the ability of analyzing and solving practical problems.

2. How to integrate mathematical modeling ideas into higher mathematics teaching

Many concepts and theorems in higher mathematics are derived from practical problems. Through a certain kind of practical problems, we can abstract the symbols and tools in mathematics to better solve such problems. So teaching should be problem-oriented for guiding students to think about and sum up by observing some phenomena. Here are some practical examples to explain how to integrate mathematical modeling ideas into the teaching of higher mathematics.

For example, when explaining the concept of derivative, teachers should not limit the definition of derivative directly, the derivative represents the slope of the tangent, etc. We should instill a mathematical idea into the students, and express the derivative directly as the rate of change, so that students can intuitively touch and understand an unknown concept, and then let the students consider what kind of phenomenon the rate of change represents, such as the speed of the temperature changes with time when the object cools in real life and the speed of displacement of the object changes with time, etc. Through this way students can experience the rate of change in real life for better understanding what the original derivative is describing. When explaining the concept of the integral, we must let the students understand the actual background, the story involved, the purpose, and the meaning of the points, and use it to solve problems in real life and bring math and life closer. You can ask students how to find the various geometric areas, such as how to find the area of the ellipse. Give everyone a little time to think about it and find out the differences between them and the regular graphics through discussions, how to deal with this problem from constants to variables. You can also think about the mathematical problems of variables, such as physical constant force work to variable force work, uniform linear motion to variable speed linear motion. Through analysis, we find that the common problems from different backgrounds ultimately come down to a kind of problems. Whether it can be solved with better methods or tools? This avoids a lot of repetitive work, which leads to the core content of concept theorem. The concept of definite integral is given logically from concrete to Abstract.

When explaining the differential mean value theorem, most teachers only explain how to

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construct auxiliary functions and prove the theorem from a geometric point of view. In fact, it can also be explained in terms of its physical practical significance, such as the average speed is equal to the instantaneous speed at a certain moment. For the Lagrangian median theorem, for instance, a tourist starts at 9:00 from the foot of the mountain and climbs to the top of the mountain at 14:00. From the top of the mountain at 9:00 the next morning, it reaches the foot of the mountain at 14:00. Question: Can you find the same speed at a certain moment in the process of going up and down the mountain? This readily comprehensible example inspires students to think. Both make different values for the cultivation of students' ability. The former highlights the understanding of the theorem proficiency and the improvement of the proof ability. The latter emphasizes the understanding of the theorem's connotation and the improvement of the application ability. It should be based on the actual situation of students and has different emphasis on different teaching methods. In an easy-to-understand way, materials, examples, and calculus, the premise requires teachers to prepare for the course. The above example can also be used to explain the median theorem of a unary function, for example: Someone starts from the mountain at 8 am on the first day and reaches the top at 14 o'clock. At 8:00 the next morning, I will return from the top of the mountain to follow the original route, and arrive at the starting point at 14:00. Question: Is there such a position in the two-day trip that the person is at the same time passing through this position? A mathematical model can be established for this practical problem: a known continuous function sum $f(x)$ and $g(x)$, where $x \in [a,b]$, and $f(a)=8, f(b)=14$ and $g(a)=14, g(b)=8$, verify that there is a little $x_0 \in [a,b]$, so that $f(x_0) = g(x_0)$.

As a basic subject, higher mathematics should weaken the proof process of the theorem and teach students to understand the content of the theorem with intuitive geometric interpretation to reduce the burden of students' thinking. For instance, when explaining the homogeneous equations of differential equations, we can ask students what the shape of the mirror or the mirror of the car's headlights is and why should we design it like this and change it to other shapes. These are practical problems in reality that can arouse students' strong curiosity and active classroom atmosphere. When explaining the limitation, you can ask the question: how many times does the minute hand and the hour hand coincide in the day? This kind of problem is often encountered in the life of the students. The teacher can leave it to the students to think about it. This phenomenon is common, so they should be interested in thinking about ways to solve such problems. The whole modeling process is a process of continuously exploring, innovation, improvement, and optimization, allowing students to become the main body and to train students' ability of observation, unity and cooperation.

3. Incorporating mathematical modeling case analysis into teaching

Case selection should be exquisite and appropriate, practical and practical, and students should be interested in it. It is described in easy-to-understand language. The language of the class is humorous and funny. The content is full of life and culture, which makes the mathematics restore the original human and life color. The following examples are given in the analysis of the teaching. For example, the introduction of the "object cooling model" when explaining the first-order ordinary differential equation.

Case 1 A glass of 100 ° C boiling water was placed in the ambient temperature of 20°C to observe the change of water temperature during the cooling process.

Analysis: According to Newton's law of cooling, the rate of change of the object's temperature $T(t)$ is proportional to the difference between the temperature of the object and the environment.

Let $T = T(t)$, so $\frac{dT}{dt} = -k \ (T - 20) \ (k > 0)$, the previously studied equation. Can we solve the

functional relationship with this equation $T(t)$ If it can be solved, what is the solution of the equation? How to solve it? Through these problems we can first introduce the concept of

differential equation: containing the unknown function T and its first derivative $\frac{dT}{dt}$ The equation is called the first-order differential equation. Model Solving: Determine the relationship between T with time t. It needs to be solved from the equation T. How to solve the differential equation? Rewrite the equation $\frac{1}{T-20}dt = -k\,dt$, so that the variable T and t are separated, and the two sides integrate at the same time $\int \frac{1}{T-20}dt = \int -k\,dt$, that is $\ln|T-20| = -kt + \ln C$, then $T = Ce^{-kt} + 20$.

Thus, the temperature of the water during the cooling process can be obtained at any time without measurement. Then let the students think about: What aspects can this model be applied to? For example, the forensic inside the detective novel identifies someone who died a few hours ago. Such models are easy to understand, and students often see this in TV dramas and novels. Therefore, it is easy for students to resonate and stimulate students to explore and think, so that students can experience the fun of solving practical problems with mathematical theory, consolidate the ideas and methods of mathematical modeling, and cultivate their problems in the future. The way of thinking that rationally thinks about problems.

Case 2 The shape of the searchlight.

Students can ask such questions in class: What is the shape of the mirror on the headlight of the car? Why is it so designed? Can you explain it with the knowledge of the calculus you learned? The advantage of the searchlight is: by point source The emitted light can be reflected out in parallel when passing through the mirror, so it illuminates farther. As shown in Fig. 1, the mirror of the concentrating mirror of the searchlight is formed by rotating the curve L around the axis X one turn, and the point source is O, the curve $L: y = y(x)$, let M(x, y) be any point on the curve, MT is tangent, and the slope is y'. The mirror shape of the searchlight is analyzed as follows, because $\angle$ OMN=$\angle$NMR, so $\tan \angle OMN = \tan \angle NMR$, $yy'^2 + 2xy' - y = 0$, $y' = -\frac{x}{y} \pm \sqrt{1 + \left(\frac{x}{y}\right)^2}$ can be obtained from the tangential angle formula and the result is $y^2 = 2C\left(x + \frac{C}{2}\right)$. Then the equation of the mirror is $y^2 + z^2 = 2C\left(x + \frac{C}{2}\right)$. The figure 1 is a typical parabola rotating around the X-axis, which represents the equation of the shape of the searchlight.

Through the explanation of this case, students can not only make students intuitively impression the Abstract differential equations, but also make students realize that the differential equations are closely related to the actual teaching content, so students can understand and grasp the learning more easily. Content and turn it into reality.

Case 3 Hungry wolf chasing rabbit problem.

There is a rabbit, a wolf, and the rabbit is located 100m west of the wolf. It is assumed that the rabbit and the wolf simultaneously find each other and start together. The rabbit runs to the nest 60m north of the wolf, and the wolf is chasing the rabbit. Rabbits and wolves are known to run at a constant speed and the speed of the wolf is twice that of a rabbit. Then ask students whether the wolf can catch the rabbit for a delicious meal?

Analysis: Assuming that the wolf always pursues in the direction of the rabbit, in order to study whether the wolf can catch up with the rabbit, you can first consider the curve of the wolf chasing the rabbit. Established a right-angle reference system, with the rabbit as the origin O, the east-west, north-south direction is the x-axis, the y-axis, A is the initial position of the wolf, and B is the rabbit's nest.
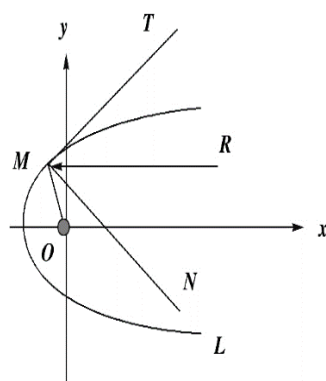
Fig.1 Reflection result

Tab. 1 Teaching model case

Teaching content	Teaching model case
The function of the limit	the area of cutting the circle, the number of coincidences of the hour and minute hands, continuous compound interest (study loans, housing loans), plant growth, population growth, bacterial reproduction, economic growth, environmental pollution indicators and other models, driving and drinking problems , the "charm" problem of illegal pyramid schemes, reasonable tax avoidance
Derivative	shape design of cans , mobile phone manufacturer pricing issues, optimization model (commodity storage cost optimization, batch purchase cycle, disk maximum storage, traffic light in traffic management, the water-saving problem of washing machine)
Differential	infectious disease model, distribution of drug components in the body
Integral	calculation of the second cosmic velocity, coverage area of communication satellites, planetary orbital problems, setting of yellow light flashing time of traffic information, prediction of natural gas production
ordinary differential equation	The shape of the mirror , the cooling model of water (forensic identification), the process of snow melting process, the law of falling parachute movement, the identification of fake painting, the treatment of nuclear waste, the decay of radioactive elements, the prevalence of infectious diseases, mortgage problems, Hungry wolf chasing rabbit problem, missile tracking problem
Series	The number of turtles and rabbits, paradox, investment costs

Let the rabbit's speed be v, and the wolf's pursuit curve be $y = y(x)$. Then according to the tangential direction of a certain point p(x, y) of the wolf, the distance between the rabbit and the wolf in the same time is twice that of the rabbit, so that two equations are listed,

$$y' = -\frac{vt-y}{x}, \quad 2vt = \int_x^{100} \sqrt{1+y'^2}\, dx \quad \text{and then} \quad 2(y - xy') = \int_x^{100} \sqrt{1+y'^2}\, dx,\quad \text{that is}$$

$$2xy'' = \sqrt{1+y'^2},$$ the initial condition is $y(100) = 0, y'(100) = 0$, combined with the computer using the mathematical software MATLAB solution, $y = \frac{x^{\frac{3}{2}}}{30} - 10x^{\frac{1}{2}} + \frac{200}{3}$, when x = 0,

$y = \frac{200}{3} > 60$, it shows that the wolf can not catch up with the rabbit, can only return to the feathers.

If the question is changed to the speed of the wolf is three times that of the rabbit, what is it like? Or what is the r-fold? Let the students analyze and discuss under the class. This example can not only understand high-order differential equations, but also incorporate mathematical modeling. At the same time, it can be extended to military missile pursuit and interception issues. Table 1 is a common example of a teaching model.

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4. Conclusion

In the background of the "Internet +" information age, we have adapted to the pace of the development of the times, and incorporated mathematical modeling ideas from the teaching content and teaching methods in the process of higher mathematics teaching, so that students no longer mechanically learn advanced mathematics. Instead, we learn mathematics from actual cases, learn mathematics from well-designed micro-curricular resources, combine traditional teaching with micro-curricular resources and multimedia, and constantly promote the reform and development of higher mathematics teaching.

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